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THE EFFECTS OF BUBBLE TRANSLATION ON VAPOR BUBBLE GROWTH IN A SUPERHEATED LIQUID

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Abstract—A theoretical analysis of vapor bubble growth in a uniformly superheated liquid has been carried out to determine the effects of translational motion of the bubble on the bubble growth rate. Assuming potential flow in the region surrounding the bubble the appropriate convective diffusion equation is solved by means of a new similarity transformation. The results of the theoretical analysis are compared with available experimental data and with analyses of the limiting cases of no bubble translation and quasi steady state bubble growth. The analysis is shown to reduce to the Plesset and Zwick or Scriven analysis for stationary growing bubbles. The effects of translation are found to be significant when the translational velocity is sufficiently high at moderate Jakob numbers, but for high Jakob numbers radial convection predominates and translation has little effect on the growth rates. The analysis predicts results in good agreement with experimental data available in the literature.

NOMENCLATURE

a,	coefficient in equation (33);
A,	$= U_\infty/R$, variable in several equations [s^{-1}];
B,	$= \dfrac{1}{R}\dfrac{dR}{dt}$, variable in several equations, [s^{-1}];
C_1, C_2,	integration constants;
C_L,	liquid phase heat capacity [cal/g°C];
F,	functional relationship;
g,	gravitational acceleration constant [cm/s^2];
$\bar{h}$,	average heat transfer coefficient [cal/scm^2 °C]
k,	thermal conductivity [cal/scm°C];
N_{Ja},	$= \dfrac{C_L(T_\infty - T_s)}{\lambda}\dfrac{\rho_L}{\rho_V}$, Jakob number [dimensionless];
N_{Nu},	$= 2\bar{h}R/k$, Nusselt number [dimensionless];
N_{Pe},	$= 3U_\infty R/2\alpha$ Péclét number [dimensionless];
p,	dummy variable of integration;
r,	radial position [cm];
R,	bubble radius [cm];
s,	dummy variable of integration;
t,	time [s];
T,	temperature [°C];
T^*,	$= 3U_\infty t/2R$, dimensionless time;
U_r,	radial velocity [cm/s];
U_θ,	tangential velocity [cm/s];
U_∞,	translational velocity of center of bubble [cm/s];
y,	distance from bubble surface [cm].

Greek letters

α,	thermal diffusivity [cm^2/s];
β, β',	growth constants;
δ,	thermal boundary layer thickness [cm];
ε,	$= \delta^2$, variable in several equations [cm^2];
η,	similarity variable;
θ,	angle [radians];

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λ, heat of vaporization [cal/g];
ν, kinematic viscosity [cm^2/s];
ξ, dummy variable of integration;
ρ, density [g/cm^3];
σ, surface tension [dynes/cm];
τ, time [s];
ϕ, function defined by equation (24);
ω, function in equation (34) and tabulated in [11].

Subscripts
L, refers to the liquid phase;
s, refers to a saturation condition;
0, refers to an initial condition;
V, refers to the vapor phase;
∞, refers to a value at great distance from the bubble.

INTRODUCTION

PROBLEMS of bubble growth in a superheated (or supersaturated) liquid, for the case involving no translational motion of the center of the fluid sphere relative to the surrounding continuous phase, have been studied by Plesset and Zwick [1, 2], Forster and Zuber [3], Scriven [4], Barlow and Langlois [5] and Waldman and Houghton [6]. In 1966 Goodrich [7] provided yet another analysis of the phase growth problem, and Bankoff [8] analyzed and reviewed the literature related to these problems. It can be concluded that diffusion-controlled bubble and droplet growth is reasonably well treated provided that the spherical fluid mass does not undergo translation.

As indicated by Sideman's summary [9] of transfer coefficient equations for moving but nongrowing droplets a considerable amount of work has been done related to constant volume drops. Most of the research on transfer coefficients for the region outside of a drop of constant size is restricted to steady state conditions, but Ruckenstein [10] proposed a method, based on a new similarity transformation, for the analysis of some problems of heat or mass transfer under unsteady conditions. The method was applied to a moving fluid spherical bubble of constant size by Ruckenstein [11], and Ruckenstein and Constantinescu [12] applied it to solve the problem of mass transfer to a drop growing at the tip of a capillary tube through which liquid is fed to the drop.

Perhaps the first attempt at a fundamental analysis of the problem of simultaneous translation and diffusion-controlled bubble growth or collapse is that of Tokuda *et al.* [13]. Using a potential flow model for the flow field around the bubble they developed small-time and large-time expressions for the growth rate which they solved numerically to obtain the radius as a function of time. They found that translation greatly increases growth rates over those predicted for stationary growing bubbles.

It is the purpose of the present paper to show that a similarity transformation of the type proposed in [10] and [11] may be useful in the analysis of diffusion-controlled moving bubble growth.

PROBLEM FORMULATION

We shall consider the growth of a translating spherical vapor bubble in a single-component uniformly superheated liquid of great extent compared with the volume of the bubble. When a vapor bubble is generated at a heated wall, such as occurs in nucleate boiling, simultaneous translation and growth (or collapse) occur after the bubble departs. Vapor bubble growth arising from homogeneous nucleation has been studied by Dergarabedian [14], Hooper and Abdelmessih [15], Kosky [16] and Florschuetz *et al.* [17]. Dergarabedian generated vapor bubbles in the bulk of slightly superheated water by infrared heating, and the latter investigators generated bubbles by suddenly reducing the pressure on water initially at a uniform temperature. All of these investigators reported bubble radii as a function of time and were primarily interested in radial growth rates. With the exception of Florschuetz *et al.* the authors largely ignored the possible effects of translation in their interpretation and analysis, but because of the buoyant force acting on a bubble translation occurs if the bubble is not attached to a solid surface of support.

The experiments of Florschuetz *et al.* were designed to assess the effects of translation on the growth rates of free vapor bubbles in uniformly superheated liquids. Using a drop tower they made measurements under zero gravity conditions as well as under normal conditions. Their results clearly indicate that bubble translation has an appreciable effect on growth rates.

Darby [18] measured vapor bubble growth rates for moving bubbles following detachment after heterogeneous nucleation on a solid surface. As in Dergarabedian's study superheated liquid was maintained by infrared heating. The data show some evidence that translation increases the growth rate, and Darby correlated his data for water and Freon 113 by means of an empirical equation.

It should be noted that, except for the large vapor slugs developed at large times in Kosky's experiments and for some of the larger bubbles in the normal condition runs of Florschuetz and his coworkers, spherical bubbles were involved in all of the experimental investigations.

If it is assumed that at time $t = 0$ a spherical vapor bubble is either injected into a uniformly superheated liquid or begins to grow in the liquid due to a sudden decrease of pressure on the system, then rises through the superheated liquid, the temperature field around the bubble satisfies the energy equation

$$\frac{\partial T}{\partial t} + U_r\frac{\partial T}{\partial r} + \frac{U_\theta}{r}\frac{\partial T}{\partial \theta} = \alpha\frac{1}{r^2}\frac{\partial}{\partial r}\left(r^2\frac{\partial T}{\partial r}\right). \tag{1}$$

The temperature field is considered to be axisymmetric and molecular conduction in the direction tangent to the bubble surface is assumed to be negligible compared with the convective transport in that direction. For sufficiently large Péclét numbers the thickness of the region in which all appreciable variation of temperature occurs is small and equation (1) may be approximated by

$$\frac{\partial T}{\partial t} + U_r\frac{\partial T}{\partial r} + \frac{U_\theta}{r}\frac{\partial T}{\partial \theta} = \alpha\frac{\partial^2 T}{\partial r^2}. \tag{2}$$

The boundary conditions and the initial condition are

(i) $T = T_\infty$ as $r \to \infty$
(ii) $T = T_s$ at $r = R$
(iii) $T = T_\infty$ at $t = 0$

Boundary condition (ii) arises by assuming that the interior of the bubble is at the saturation temperature.

If the center of the bubble moves at velocity U_∞ relative to stationary coordinates and the flow field around the bubble is approximated by potential flow, the velocity components are given by

$$U_r = -U_\infty\left(1 - \frac{R^3}{r^3}\right)\cos\theta + \frac{R^2}{r^2}\frac{dR}{dt} \tag{3}$$

$$U_\theta = U_\infty\left(1 + \frac{R^3}{2r^3}\right)\sin\theta. \tag{4}$$

Since we are interested in the temperature distribution in the region near the bubble surface, i.e. $y \ll R$ where $y = r - R$, the velocity components in this region can be approximated by expanding the radius-dependent terms in y/R to give, for small y/R

$$U_r = -3U_\infty\frac{y}{R}\cos\theta + \frac{dR}{dt}\left(1 - \frac{2y}{R}\right) \tag{5}$$

$$\frac{U_\theta}{r} = \frac{3}{2}\frac{U_\infty}{R}\sin\theta. \tag{6}$$

Introducing these velocity expressions in the convective diffusion equation, equation (2), one obtains

$$\frac{\partial T}{\partial t} + \left[\left(1 - \frac{2y}{R}\right)\frac{dR}{dt} - 3U_\infty\frac{y}{R}\cos\theta\right]\frac{\partial T}{\partial r} + \frac{3}{2}\frac{U_\infty}{R}\sin\theta\frac{\partial T}{\partial\theta} = \alpha\frac{\partial^2 T}{\partial r^2}. \tag{7}$$

In addition to the boundary conditions and the initial condition discussed above the radius must satisfy the relation between the bubble wall velocity $\mathrm{d}R/\mathrm{d}t$ and the rate of heat transfer to the bubble obtained by writing an energy balance on the bubble surface, i.e.

$$\frac{\mathrm{d}R}{\mathrm{d}t} = \frac{k}{2\rho_v\lambda}\int_0^\pi \left(\frac{\partial T}{\partial y}\right)_{y=0} \sin\theta\,\mathrm{d}\theta \tag{8}$$

with initial condition $R = R_0$ at $t = t_0$.

SOLUTION

Introducing new variables $y = r - R$ and $\tau = t$, equation (7) transforms to

$$\frac{\partial T}{\partial \tau} - y\left(3\,A\cos\theta + \frac{2}{R}\frac{\mathrm{d}R}{\mathrm{d}\tau}\right)\frac{\partial T}{\partial y} + \frac{3}{2}A\sin\theta\frac{\partial T}{\partial \theta} = \alpha\frac{\partial^2 T}{\partial y^2} \tag{9}$$

where $A = A(\tau) = U_\infty(R)/R$. It is assumed that the bubble rise velocity U_∞ is a function only of the bubble radius and the physical properties of the system.

Equation (9) can be solved by the method proposed by Ruckenstein [10], introducing the similarity variable

$$\eta = y/\delta(\theta, \tau)$$

and assuming that $T = T(\eta)$. Equation (9) transforms to

$$\alpha\frac{\mathrm{d}^2T}{\mathrm{d}\eta^2} + \eta\frac{\mathrm{d}T}{\mathrm{d}\eta}\left(\frac{1}{2}\frac{\partial\delta^2}{\partial\tau} + 3\,A\,\delta^2\cos\theta + 2\,B\,\delta^2 + \frac{3}{4}A\sin\theta\frac{\partial\delta^2}{\partial\theta}\right) = 0 \tag{10}$$

where

$$B = B(\tau) = \frac{1}{R}\frac{\mathrm{d}R}{\mathrm{d}\tau}.$$

If the temperature is to be a function of η only, the term in brackets in equation (10) can be set equal to a constant. A convenient choice of the constant is 2α and this choice leads to the following equations that must be solved to obtain the temperature distribution.

$$\frac{\mathrm{d}^2T}{\mathrm{d}\eta^2} + 2\eta\frac{\mathrm{d}T}{\mathrm{d}\eta} = 0 \tag{11}$$

and

$$\frac{\partial\varepsilon}{\partial\tau} + (6\,A\cos\theta + 4B)\varepsilon + \frac{3}{2}A\sin\theta\frac{\partial\varepsilon}{\partial\theta} = 4\alpha \tag{12}$$

where $\varepsilon \equiv \delta^2$.

The boundary conditions become

(i) $T = T_\infty$ as $\eta \to \infty$
(ii) $T = T_s$ at $\eta = 0$.

The well-known solution of equation (11) is

$$\frac{T - T_\infty}{T_s - T_\infty} = \operatorname{erfc}\left[y/\delta(\theta, \tau)\right]. \tag{13}$$

Equation (12) can be solved by using the method of characteristics. The characteristic system of equations which may be attached to it is

$$\frac{\mathrm{d}\tau}{1} = \frac{\mathrm{d}\theta}{\frac{3}{2}A\sin\theta} = \frac{\mathrm{d}\varepsilon}{4\alpha - (6\,A\cos\theta + 4B)\varepsilon}. \tag{14}$$

The pair of equations obtained from the characteristic system is

$$\frac{3}{2}A(\tau)\,\mathrm{d}\tau = \frac{\mathrm{d}\theta}{\sin\theta} \tag{15}$$

and

$$\frac{\mathrm{d}\varepsilon}{\mathrm{d}\tau} \quad [6A(\tau)\cos\theta + 4B(\tau)]\,\varepsilon = 4\alpha. \tag{16}$$

The solution of equation (15) is

$$C_1 + \int^\tau A(s)\,\mathrm{d}s = \frac{2}{3}\ln\tan(\theta/2). \tag{17}$$

To solve equation (16) it is first necessary to express $\cos\theta$ as a function of τ and C_1 by means of equation (17). This is done by using the trigonometric identity

$$\cos\theta = \frac{1 - \tan^2(\theta/2)}{1 + \tan^2(\theta/2)} \tag{18}$$

and by eliminating $\tan\theta/2$ between equations (17) and (18). Substituting the resulting expression for $\cos\theta$ in equation (16) and integrating one obtains.

$$\varepsilon \exp\left\{\int^{\tau}\left[6A(\xi)\frac{1 - \exp[3C_1 + 3\int^{\xi}A(s)\mathrm{d}s]}{1 + \exp[3C_1 + 3\int^{\xi}A(s)\mathrm{d}s]} + 4B(\xi)\right]\mathrm{d}\xi\right\} =$$
$$C_2 + 4\alpha\int^{\tau}\exp\left\{\int^{p}\left[6A(\xi)\frac{1 - \exp[3C_1 + 3\int^{\xi}A(s)\mathrm{d}s]}{1 + \exp[3C_1 + 3\int^{\xi}A(s)\mathrm{d}s]} + 4B(\xi)\right]\mathrm{d}\xi\right\}\mathrm{d}p. \tag{19}$$

The general solution of equation (12) has the form $C_2 = F(C_1)$. Consequently we can rewrite equation (19), using equations (17) and (18), to give

$$\varepsilon\exp\left\{\int^{\tau}\left[6A(\xi)\frac{1 - (\tan^2\theta/2)\exp(3\int_{\tau}^{\xi}A(s)\mathrm{d}s)}{1 + (\tan^2\theta/2)\exp(3\int_{\tau}^{\xi}A(s)\mathrm{d}s)} + 4B(\xi)\right]\mathrm{d}\xi\right\} -$$
$$4\alpha\int^{\tau}\exp\left\{\int^{p}\left[6A(\xi)\frac{1 - (\tan^2\theta/2)\exp(3\int_{\tau}^{\xi}A(s)\mathrm{d}s)}{1 + (\tan^2\theta/2)\exp(3\int_{\tau}^{\xi}A(s)\mathrm{d}s)} + 4B(\xi)\right]\mathrm{d}\xi\right\}\mathrm{d}p =$$
$$F(\tfrac{2}{3}\ln\tan\theta/2 - \int^{\tau}A(s)\mathrm{d}s). \tag{20}$$

The form of the function F will be determined by using the initial condition, $\varepsilon = 0$ at $\tau = 0$. This yields

$$F(\tfrac{2}{3}\ln\tan\theta/2 - \int^{0}A(s)\mathrm{d}s) =$$
$$-4\alpha\int^{0}\exp\left\{\int^{p}\left[6A(\xi)\frac{1 - (\tan^2\theta/2)\exp(3\int_{0}^{\xi}A(s)\mathrm{d}s)}{1 + (\tan^2\theta/2)\exp(3\int_{0}^{\xi}A(s)\mathrm{d}s)} + 4B(\xi)\right]\mathrm{d}\xi\right\}\mathrm{d}p. \tag{21}$$

Consequently,

$$F(\tfrac{2}{3}\ln\tan\theta/2 - \int^{\tau}A(s)\mathrm{d}s) =$$
$$-4\alpha\int^{0}\exp\left\{\int^{p}\left[6A(\xi)\frac{1 - (\tan^2\theta/2)\exp(3\int_{\tau}^{\xi}A(s)\mathrm{d}s)}{1 + (\tan^2\theta/2)\exp(3\int_{\tau}^{\xi}A(s)\mathrm{d}s)} + 4B(\xi)\right]\mathrm{d}\xi\right\}\mathrm{d}p \tag{22}$$

and, finally

$$\varepsilon = 4\alpha \int_0^\tau \exp\left[\int_\tau^p \phi(\theta, \xi, \tau)\mathrm{d}\xi\right]\mathrm{d}p \tag{23}$$

where

$$\phi(\theta, \xi, \tau) = 6A(\xi)\frac{1 - (\tan^2\theta/2)\exp\left(3\int_\tau^\xi A(s)\mathrm{d}s\right)}{1 + (\tan^2\theta/2)\exp\left(3\int_\tau^\xi A(s)\mathrm{d}s\right)} + 4B(\xi). \tag{24}$$

Thus the bubble growth rate $\mathrm{d}R/\mathrm{d}\tau$ can be obtained from the energy balance, equation (8), by differentiating equation (13), evaluating the temperature gradient at $r = R$ and substituting the result in equation (8) to give

$$\frac{\mathrm{d}R}{\mathrm{d}\tau} = N_{Ja}\frac{\alpha}{\sqrt{(\pi)}}\int_0^\pi \frac{\sin\theta\,\mathrm{d}\theta}{\sqrt{[\varepsilon(\theta,\tau)]}} \tag{25}$$

where the Jakob number N_{Ja} is defined by

$$N_{Ja} \equiv \frac{C_L(T_\infty - T_s)}{\lambda}\frac{\rho_L}{\rho_V}, \tag{26}$$

and $\varepsilon(\theta,\tau)$ is given by equation (23).

Integrating equation (25) one obtains the equation for the bubble radius as a function of time

$$R(\tau) = R_0 + \frac{N_{Ja}}{2}\sqrt{\left(\frac{\alpha}{\pi}\right)}\times$$

$$\int_0^\tau\int_0^\pi \frac{\sin\theta\,\mathrm{d}\theta}{\left\{\int_0^t \exp\left[\int_t^p\left[6A(\xi)\frac{1 - (\tan^2\theta/2)\exp\left(3\int_t^\xi A(s)\mathrm{d}s\right)}{1 + (\tan^2\theta/2)\exp\left(3\int_t^\xi A(s)\mathrm{d}s\right)} + 4B(\xi)\right]\mathrm{d}\xi\right]\mathrm{d}p\right\}^{\frac{1}{2}}}\mathrm{d}t. \tag{27}$$

Equation (27) reduces to a much simpler and well-known result when $U_\infty = 0$ and $R_0 = 0$. In this case Scriven's method of solution can be used to determine the bubble radius and the growth rate $\mathrm{d}R/\mathrm{d}\tau$ analytically. Assuming

$$R = 2\beta\sqrt{(\alpha\tau)}, \tag{28}$$

then

$$\frac{\mathrm{d}R}{\mathrm{d}\tau} = \beta\sqrt{\left(\frac{\alpha}{\tau}\right)}. \tag{29}$$

Since $U_\infty = 0$ we have $A(\tau) = 0$, and $B(\tau) = 1/\tau$. Equation (25) can then be integrated analytically to give the result obtained by Plesset and Zwick and by Scriven:

$$\frac{\mathrm{d}R}{\mathrm{d}\tau} = N_{Ja}\sqrt{\left(\frac{3\alpha}{\pi\tau}\right)} \tag{30}$$

from which one obtains the Plesset and Zwick value of the growth constant

$$\beta = \sqrt{(3/\pi)}\,N_{Ja}. \tag{31}$$

It should be pointed out that when the initial radius is nonzero owing to the nonlinear character of the integral equation (27), the solution of the equation does not have the form suggested by intuition,

$$R = R_0 + 2\beta'\sqrt{(\alpha\tau)}. \tag{32}$$

Another type of solution, differing from that of Scriven, must be found. Indeed, using equation (32) for evaluating the integral in equation (25) one obtains

$$\frac{\mathrm{d}R}{\mathrm{d}\tau} = \beta'\sqrt{\left(\frac{\alpha}{\tau}\right)} = N_{Ja}\left(\sqrt{\frac{\alpha}{\pi}}\right)\frac{(a + \tau^{\frac{1}{2}})^2}{[a^4\tau + \frac{8}{3}a^3\tau^{\frac{3}{2}} + 3a^2\tau^2 + \frac{8}{5}a\tau^{\frac{5}{2}} + (\tau^3/3)]^{\frac{1}{2}}} \tag{33}$$

where $a \equiv R_0/2\beta'\sqrt{(\alpha)}$. Equation (33) invalidates the original assumption that $\beta' =$ constant, except for very large times, for which the equation reduces to $\beta' = N_{Ja}\sqrt{(3/\pi)}$.

When $U_\infty \neq 0$ the functions $A(\xi)$ and $B(\xi)$, which are functions of the bubble radius, depend upon time in an unknown manner, so a numerical method is required to solve the nonlinear integral equation, equation (27). An iterative procedure involving a method of successive approximations was used to solve the equation. To obtain trial values of the radius for the iterative numerical solution the approximation,

$$R = R_0 + 2N_{Ja}\sqrt{(3\alpha\tau/\pi)},$$

was used to begin computations. For later times (after the first few time increments) rapid convergence of the iteration scheme was achieved by obtaining trial values of R for each new time increment by linear extrapolation of the values from two previously calculated increments. Simpson's rule was used in the numerical integration schemes, and the effects of the sizes of the time increment and the angle increment were established. Solutions were found to be quite insensitive to the size of the angle increment, but because of the multiple integrals involved (with respect to time) the time increments required for accuracy and stability of the calculations were of the order of 10^{-5}–10^{-4} s for the results discussed below.

LIMITING CASES

Qualitative information about the time-dependent thermal boundary layer thickness and about the relative importance of translation compared with radial motion can be obtained from equations (23) and (24). The thermal boundary layer thickness decreases as the function $\phi(\theta, \xi, \tau)$, defined by equation (24) as the sum of translational and radial convection terms, increases. The radial growth is a function of the Jakob number and the translational convection can be associated with a Péclét number ($N_{Pe} = 3RU_\infty 2\alpha$). An increase in either of these terms will produce a thinning of the thermal boundary layer.

If

$$4B(\xi) \gg 6A(\xi)\frac{1 - \exp[3\int_\tau^\xi A(s)\mathrm{d}s]\tan^2\theta/2}{1 + \exp[3\int_\tau^\xi A(s)\mathrm{d}s]\tan^2\theta/2},$$

the effects of translation are insignificant, and radial transport of heat predominates. Provided that the Jakob number (and, therefore, the growth rate) is sufficiently large that the thin thermal boundary layer assumption is valid, the solution reduces to that of Plesset and Zwick as shown above.

When the translational convection term and the radial convection term of $\phi(\theta, \xi, \tau)$ are of the same order both modes of convection are important, and the numerical solution discussed above must be used to compute the results.

Tokuda, Yang and Clark came to the same qualitative conclusions about the thickness of the thermal boundary layer and the importance of axial versus radial convection. By writing the energy equation in a dimensionless form they showed that the thickness of the time-dependent thermal boundary layer depends upon the Jakob number and the Péclét number, and they showed that when N_{Ja} is of the order of N_{Pe}^2 both radial and axial convection terms are significant. They proceeded to solve the original energy equation by a perturbation technique and obtained numerical results.

A third possibility exists concerning the relative magnitudes of the translational and radial convection terms. When

$$6A(\xi)\frac{1-\exp[3\int_\tau^\xi A(s)\mathrm{d}s]\tan^2\theta/2}{1+\exp[3\int_\tau^\xi A(s)\mathrm{d}s]\tan^2\theta/2} \gg 4B(\xi), \text{ i.e. when } N_{Pe} \gg N_{Ja},$$

translational effects predominate, and the heat flux at each moment should be given by that for heat transfer to a bubble of constant size, provided that $\phi(\theta, \xi, \tau)$ is sufficiently large that the thin thermal boundary layer approximation is valid. The growth rate should then be obtainable by quasi steady state methods.

THE QUASI STEADY STATE APPROXIMATION

If it is assumed that at each moment of time the heat transfer coefficient is given by expressions valid for the corresponding constant radius moving bubble, the heat transfer rate may be obtained from previously developed expressions [11]. For potential flow around a constant size bubble the Nusselt number is given by

$$N_{Nu} = (N_{Pe}/\pi)^{\frac{1}{2}}\omega(T^*) \tag{34}$$

where the Nusselt number is defined by $N_{Nu} = 2\bar{h}R/k$, the heat transfer coefficient $\bar{h}$ is defined in terms of the average heat flux $\bar{q}$ over the bubble surface, i.e. $\bar{h} = \bar{q}/(T_\infty - T_s)$, and $\omega(T^*)$ is a time-dependent function calculated in [11]. The Péclét number and the dimensionless time T^* are given by

$$N_{Pe} = \tfrac{3}{2}RU_\infty/\alpha \qquad \text{and } T^* = \tfrac{3}{2}tU_\infty/R.$$

For $T^* \gg 1$, $\omega(T^*) = 4/\sqrt{(3)}$ and equation (34) becomes

$$N_{Nu} = 4(N_{Pe}/3\pi)^{\frac{1}{2}}. \tag{35}$$

The bubble growth rate is obtained from an energy balance on the bubble,

$$\frac{\mathrm{d}}{\mathrm{d}t}(\rho_v\lambda\tfrac{4}{3}\pi R^3) = 4\pi R^2\bar{h}(T_\infty - T_s). \tag{36}$$

For constant vapor density and constant heat of vaporization the growth rate equation becomes

$$\frac{\mathrm{d}R}{\mathrm{d}t} = \frac{N_{Ja}}{2}\left(\frac{3\alpha U_\infty}{2\pi R}\right)^{\frac{1}{2}}\omega(T^*) \tag{37}$$

where ω is a function of time and radius as indicated above. If the translational velocity is specified as a function of the radius, equation (37) can be integrated numerically to give the time-dependent radius.

THE TRANSLATIONAL VELOCITY

In the above analyses the translational velocity is considered to be a function of the radius, but no particular form of that functional relationship has been specified. Any appropriate equation for the translational velocity or even experimental data can be used in the numerical solutions of equations (27) and (37). Three different approaches are used for the calculations discussed below (i) a constant translational velocity, (ii) experimental data and (iii) velocities calculated for each point in time from well-known expressions that rigorously apply to nongrowing bubbles. In the latter case the inertial terms that are ignored are probably significant during a small-time period when the bubbles grow most rapidly, but for simplicity this complication is not taken into account here.

Levich [19] analyzed the drag on a spherical bubble. The analysis, which has recently been extended by Harper and Moore [20], applies to the motion of a constant radius bubble in an uncontaminated liquid for bubble Reynolds numbers greater than about 100. For bubbles larger than approximately 0·04 cm, however, Levich's equation,

$$U_\infty = gR^2/9\nu, \tag{38}$$

predicts bubble rise velocities that are too high compared with experimental data. A better expression for larger bubbles is that of Mendelson [21],

$$U_\infty = \left(\frac{\sigma}{R\rho_L} + gR\right)^{\frac{1}{2}}, \tag{39}$$

which Cole [22] has shown to be in reasonable agreement with experimental data for steam and organic vapor bubbles under nucleate boiling conditions.

RESULTS

Quantitative information about the relative importance of translation compared with radial motion was obtained by making a parametric study of the solution of equation (27). Figure 1

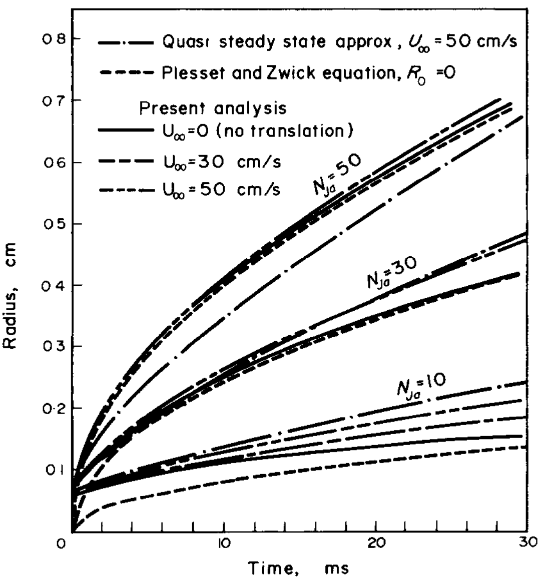

FIG. 1. The results of a parametric study of equation (27) for water at 2 atm pressure.

shows the results of such a study for various Jakob numbers and various constant translational velocities including the case of no translation. The physical properties used in the calculations correspond to steam and water at a pressure of 2 atmospheres, and an initial radius of 0·057 cm, the departure diameter for steam bubbles from a solid surface at this pressure, is assumed. The Plesset and Zwick or Scriven asymptotic solution for stationary bubbles, equation (28), (which we shall refer to as the PZS solution) is plotted for the three Jakob numbers shown. The other limit, quasi steady state growth, is also shown on Fig. 1.

For large Jakob numbers ($N_{Ja} > 50$) and at small times even relatively large translational velocities ($U_\infty = 50$ cm/s) have little effect on the growth rate because radial convection predominates. The PZS solution is in good agreement with the solutions for large Jakob numbers even though the initial radius was taken as finite for the solution of equation (27). It is not surprising that the quasi steady state approximation predicts lower growth rates for $U_\infty = 50$ cm/s and $N_{Ja} = 50$ than either equations (27) or (28), for the growth is dominated by radial convection and translation is only slightly significant in the time range shown in Fig. 1.

The quasi steady state approximation is in good agreement with the results of equation (27) for $U_\infty = 50$ cm/s and $N_{Ja} = 30$. For these conditions translational convection predominates. Again the PZS result is in good agreement with the numerical solution of equation (27) for the case of no translation.

The predicted growth curves for $N_{Ja} = 10$ show a strong effect of translation, but the quasi steady state solution for $U_\infty = 50$ cm/s and the results of equation (27) for that velocity are not in agreement. Furthermore, because of the relatively large initial radius the results of equation (28), which applies only for zero initial radius, are not in agreement with those of equation (27).

Some caution must be exercised in applying the present analysis to low Jakob numbers, for the assumption of a thin thermal boundary layer is questionable if the translational velocity is also low. To examine this point sample results for the function $\varepsilon/4\alpha$ from equation (23) corres-

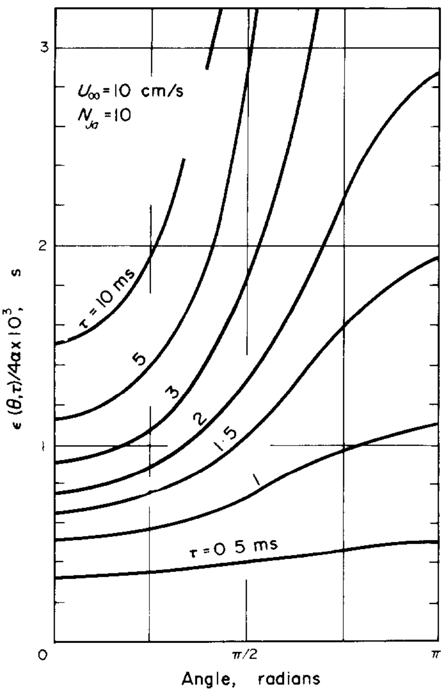

FIG. 2. The behavior of the thermal boundary layer thickness for low Jakob number and low velocity.

ponding to the parametric studies of Fig. 1 are shown in Fig. 2 for a combination of low Jakob number and low translational velocity. Even at small times the thermal boundary layer in the rear half of the bubble is relatively large for this extreme case. For the range of times shown in the figure $\delta/R < 0{\cdot}04$ at the front stagnation point, so the thin thermal boundary layer assumption is valid in this region, but at the rear stagnation point δ quickly becomes large. For larger translational velocities with $N_{Ja} = 10$ the thickening of the thermal boundary layer is less pronounced, but near $\theta = \pi$ the validity of the analysis is questionable. Even at higher Jakob numbers δ is large in the vicinity of $\theta = \pi$, but this region has very little effect on the solution as seen by examination of the integrand in equation (27).

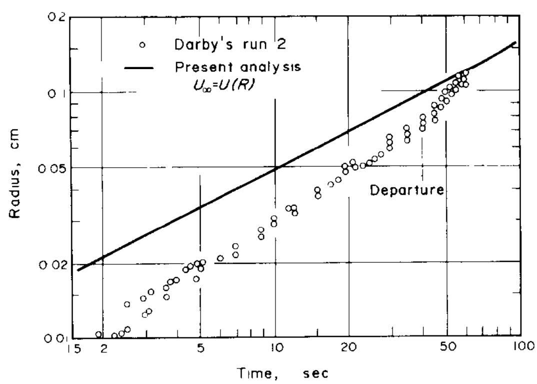

FIG. 3. A comparison of the present analysis with typical data of Darby for water.

It is possible to obtain some information on the usefulness of the present analysis for low Jakob number systems by comparing it with available experimental data. The data available for simultaneously growing and translating bubbles are in the low Jakob number range ($3 < N_{Ja} < 10$). Darby provided data on both the bubble radius as a function of time and the position of the bubble as a function of time. As the bubbles were nucleated on a solid surface the experimental growth data prior to bubble departure do not agree with the PZS solution. The results for two of Darby's runs are shown in Figs. 3 and 4, and the growth curves predicted

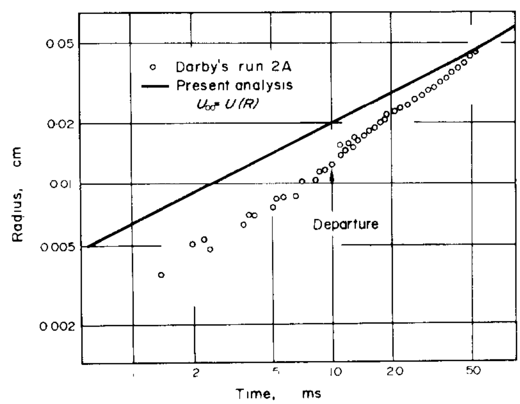

FIG. 4. A comparison of the present analysis with typical data of Darby for Freon 113.

from equation (27) are plotted on the figures. The translational velocities used were those obtained by curve-fitting the appropriate experimental data. The data for water, Fig. 3, show some effect of translation on the growth, but the predictions from equation (27) deviate only slightly from the PZS solution for stationary growing bubbles. For Freon 113 the data show (Fig. 4) little effect of translation and the results predicted using equation (27) are nearly indistinguishable from the PZS results.

The data of Florschuetz *et al.* are better suited for comparison with the present analysis, for their work involved homogeneous nucleation in the bulk of a superheated liquid, so the effects of solid boundaries and bubble departure are absent. Unfortunately they did not supply information on the bubble translational velocities as a function of time or of bubble radius, so some conjecture about these velocities must be made. They reported that the translational velocities were in the range from 0 to 40 cm/s, and therefore any assumptions made about the translation must be consistent with the upper limit of 40 cm/s.

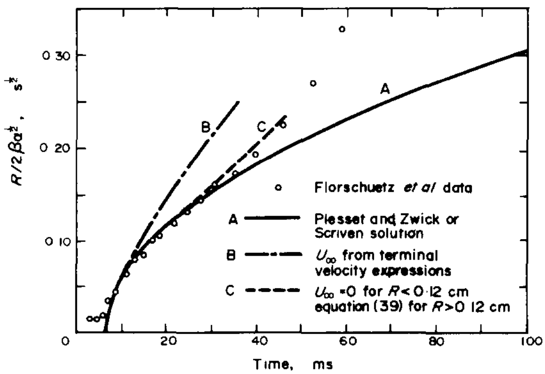

FIG. 5. A comparison of theoretical analyses with data for water.

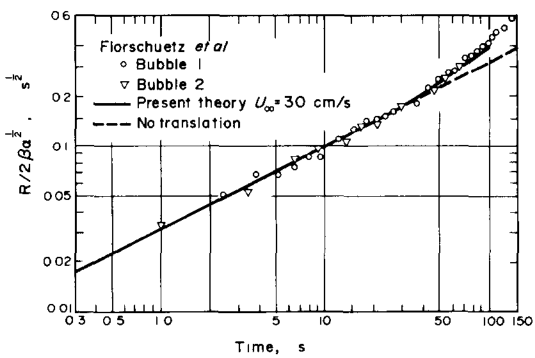

FIG. 6. A comparison of the present analysis with data for water.

Figure 5 shows a comparison among the results predicted with the present analysis (with two different policies for calculating the translational velocity), the PZS analysis and experimental data for steam bubbles of Florschuetz *et al.* If equations (38) and (39) are used to predict U_∞ over the entire range of bubble sizes the growth rate, Curve B, is considerably overpredicted. It is likely that the calculated velocities are much too high in the early phase of bubble growth. An alternate policy was to assume that translation was not significant up to a radius of 0·12 cm, and after that point U_∞ was calculated from equation (39). The result, Curve C, is in reasonably good agreement with the experimental data and the calculated velocities are in the range observed by Florschuetz and his associates. Significant deviation from the predictions for no translation, Curve A, occurs.

Parametric studies based on the conditions of Florschuetz *et al.* indicate that it is the velocity attained after the first 20 or 30 ms that has an appreciable effect on the bubble growth. Provided that the velocities calculated for the smaller bubbles are not too large translation has no effect on bubble growth at small times (t < 20 ms). Furthermore, translational velocities less than about 20 cm/s have little effect on the bubble growth at large times.

Figures 6–8 compare experimental data for water, ethanol and isopropanol, respectively, with predictions made using a constant translational velocity of 30 cm/s in equation (27) for t > 10 ms. The results of the PZS solution for no translation are also plotted on the figures. The velocity of 30 cm/s is well within the range of translational velocities reported by the investigators. The predictions are in reasonably good

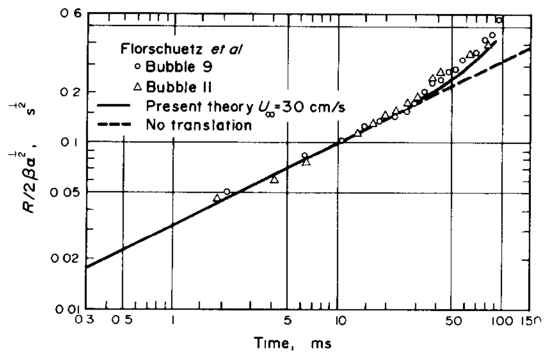

FIG. 7. A comparison of the present analysis with data for ethanol.

agreement with the experimental data for all three systems, and the deviations from the theory for stationary phase growth are correctly predicted. Similar results are obtained using equation (39) to predict the translational velocities after the first 10 ms.

To use the present analysis as a reliable predictive theory it is necessary to have better methods of predicting the translational velocity of growing bubbles, but Mendelson's equation, equation (39), appears to be satisfactory for larger bubbles.

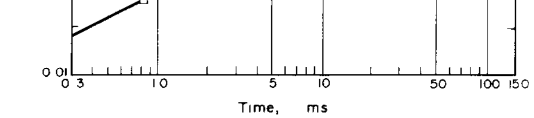

FIG. 8. A comparison of the present analysis with data for isopropanol.

CONCLUSIONS

The effects of translation on diffusion-controlled vapor bubble growth have been studied by means of a new similarity solution of the convective diffusion equation. For sufficiently large Jakob numbers ($N_{Ja} > 50$) bubble growth is not greatly affected by translation, for radial convection predominates. For moderate Jakob numbers translation can substantially increase bubble growth rates over the rates predicted for stationary bubble growth, for both translational and radial convection are important at moderate translational velocities. If the velocity of translation is sufficiently large bubble growth characteristics can be predicted by quasi steady state methods, and in the limit of no translation and small initial bubble radius the new solution reduces to the Plesset and Zwick or Scriven solution for diffusion-controlled phase growth.

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REFERENCES

1. M. S. PLESSET and S. A. ZWICK, A nonsteady heat diffusion problem with spherical symmetry, *J. Appl. Phys.* **23**, 95–98 (1952).
2. M. S. PLESSET and S. A. ZWICK, The growth of vapor bubbles in superheated liquids, *J. Appl. Phys.* **25**, 493–500 (1954).
3. H. K. FORSTER and N. ZUBER, Growth of a vapor bubble in a superheated liquid, *J. Appl. Phys.* **25**, 474–478 (1954).
4. L. E. SCRIVEN, On the dynamics of phase growth, *Chem. Engng Sci.* **10**, 1–13 (1959).
5. E. J. BARLOW and W. E. LANGLOIS, Diffusion of gas from a liquid into an expanding bubble, *IBM Jl* **6**, 329–337 (1962).
6. L. A. WALDMAN and G. HOUGHTON, Spherical phase growth in superheated liquids, *Chem. Engng Sci.* **20**, 625–636 (1965).
7. F. C. GOODRICH, On diffusion-controlled particle growth; the moving boundary problem, *J. Phys. Chem.* **70**, 3660–3665 (1966).
8. S. G. BANKOFF, *Advances in Chemical Engineering*, Vol. 6. Academic Press, New York (1966).
9. S. SIDEMAN, *ibid.*
10. E. RUCKENSTEIN, A generalized penetration theory for unsteady convective mass transfer, *Chem. Engng Sci.* **23**, 363–371 (1968).
11. E. RUCKENSTEIN, Mass transfer between a single drop and a continuous phase, *Int. J. Heat Mass Transfer* **10**, 1785–1792 (1967).
12. E. RUCKENSTEIN and D. CONSTANTINESCU, Mass or heat transfer with a change in interfacial area. I. Mass transfer in the continuous phase to a growing drop, *Int. J. Heat Mass Tranfer* **12**, 1249–1255 (1969).

13. N. TOKUDA, W. J. YANG and J. A. CLARK, Dynamics of moving gas bubbles in injection cooling, *J. Heat Transfer* **90**, 371–378 (1968).
14. P. DERGARABEDIAN, The rate of growth of vapor bubbles in superheated water, *J. Appl. Mech.* **20**, 537–545 (1953).
15. F. C. HOOPER and A. H. ABDELMESSIH, The flashing of liquids at higher superheats, *Proc. 3rd International Heat Transfer Conference*, Vol. 4, pp. 44–50 (1966).
16. P. G. KOSKY, Bubble growth measurements in uniformly superheated liquids, *Chem. Engng Sci.* **23**, 695–706 (1968).
17. L. W. FLORSCHUETZ, C. L. HENRY and A. R. KHAN, Growth rates of free vapor bubbles in liquids at uniform superheats under normal and zero gravity conditions, *Int. J. Heat Mass Transfer* **12**, 1465–1489 (1969).
18. R. DARBY, The dynamics of vapour bubbles in nucleate boiling, *Chem. Engng Sci.* **19**, 39–49 (1964).
19. V. G. LEVICH, Motion of gas bubbles at large Reynolds numbers, *Zh. Eksp. I. Teoret, Fiz.* **19**, 18 (1949).
20. J. F. HARPER and D. W. MOORE, The motion of a spherical liquid drop at high Reynolds numbers, *J. Fluid Mech.* **32**, 367–391 (1968).
21. H. D. MENDELSON, The prediction of bubble terminal velocities from wave theory, *A.I.Ch.E. Jl* **13**, 250–253 (1967).
22. R. COLE, Motion of vapor bubbles in saturated liquids, *A.I.Ch.E. Jl* **13**, 403–404 (1967).

LES EFFETS DE TRANSLATION DE BULLE SUR LA CROISSANCE DE BULLES DE VAPEUR DANS UN LIQUIDE SURCHAUFFÉ

Résumé—Une analyse théorique de la croissance de bulle de vapeur dans un liquide uniformément surchauffé a été menée pour déterminer les effets de mouvement de translation de la bulle sur sa vitesse de croissance. En supposant un écoulement potentiel dans la région voisine de la bulle, l'équation appropriée de diffusion convective est résolue à l'aid d'une nouvelle transformation de similitude. Les résultats de l'analyse théorique sont comparés aux résultats expérimentaux utilisables et aux analyses des cas limites de non-translation de la bulle et de croissance de la bulle en état quasi-stationnaire. On montre que l'analyse se réduit à celles de Plesset et Zwick ou Scriven pour des bulles croissant stationnairement. On montre que les effets de translation sont significatifs quand la vitesse de translation est suffisamment grande pour des nombres de Jakob modérés mais pour des nombres de Jakob élevés la convection radiale prédomine, et la translation a peu d'effet sur les vitesses de croissance. L'analyse prédit des résultats en bon accord avec ceux expérimentaux utilisables dans la littérature.

DIE EFFEKTE DER TRANSLATIONSBEWEGUNG DER BLASEN AUF DAS DAMPFBLASENWACHSTUM IN EINER ÜBERHITZTEN FLÜSSIGKEIT

Zusammenfassung—Eine theoretische Ableitung des Dampfblasenwachstums in einer überhitzten Flüssigkeit ist durchgeführt worden, um die Einflüsse der Translationsbewegung der Blasen auf die Blasenwachstumsraten zu bestimmen. Unter der Annahme einer Potentialströmung in der Umgebung der Blase wird die zugehörige konvektive Diffusionsgleichung mit Hilfe einer neuen Ähnlichkeitstransformation gelöst. Die Ergebnisse der theoretischen Ableitung werden mit den vorhandenen experimentellen Daten und mit den Ableitungen der Grenzfälle nämlich keine Blasentranslation und quasistationäres Blasenwachstum verglichen. Die Ableitung lässt sich auf die von Plesset und Zwick oder Scriven angegebene Analyse für das stationäre Blasenwachstum zurückführen. Die Einflüsse der Translation sind bedeutend, wenn die Translationsgeschwindigkeit bei mittleren Jakob-Zahlen gross ist; bei grossen Jakob-Zahlen überwiegt jedoch die radiale Konvektion und die Translation hat nur einen geringen Einfluss auf die Wachstumsraten. Die Ableitung liefert Ergebnisse von guter Übereinstimmung mit den vorhandenen experimentellen Daten aus der Literatur.

ВЛИЯНИЕ ПОСТУПАТЕЛЬНОГО ДВИЖЕНИЯ ПУЗЫРЬКОВ ПАРА НА ИХ РОСТ В ПЕРЕГРЕТОЙ ЖИДКОСТИ

Аннотация—Проводится теоретический анализ роста паровых пузырьков в равномерно перегретой жидкости с целью определения влияния поступательного движения пузырьков на скорость их роста. Соответствующие уравнения конвективной диффузии решаются с помощью нового автомодельного преобразования при допущении о существовании потенциальногопотока вблизи пузырька. Результаты теоретического анализа сравниваются с опубликованными экспериментальными данными, а также с аналитическими решениями для предельных случаев отсутствия движения пузырька и квазистационарного роста пузырей. Показано, что для стационарного роста пузырей анализ сводится к анализу Плессе и Цвика или Скривена. Установлено, что влияние поступательного движения существенно, когда его скорость достаточно велика при умеренных числах Якоба; при больших же числах Якоба преобладает радиальная конвекция, а поступательное движение не оказывает значительного влияния на скорость роста. Результаты анализа хорошо согласуются с имеющимися в литературе экспериментальными данными.